EFFECTIVE COMMUNICATION AND AFFECTIVE DOMAIN IN THE LEARNING OF MATHEMATICS

COMUNICACIÓN EFECTIVA Y DOMINIO AFECTIVO EN EL APRENDIZAJE DE LAS MATEMÁTICAS

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ABSTRACT

Mathematics has been and will continue to be a rigorous and unavoidable foundation in a large number of disciplines and specialties, which shows the need for its inclusion and its potential in many curricular proposals within academic training at all levels. However, various authors refer to it as the least popular of the disciplines in almost any curriculum. This lack of popularity and, consequently, the difficulties that arise both in their learning and in their teaching, has multiple reasons. The truth is that there is a significant student aversion to mathematics, thus generating low levels of performance in this area of knowledge. In this article, two elements considered decisive for the learning of mathematics will be exposed. First, effective communication and language as the focus for teachers and students to reach agreements on the meanings of shared mathematical objects and, second, the affective domain, related to beliefs, attitudes, and emotions, which they determine the subjective assessment that each individual shows when faced with learning processes related to mathematics and that they are precisely responsible for the reactions, more negative than positive, that they express.

KEYWORDS: effective communication – affective domain – learning - mathematics.

RESUMEN:

La matemática ha sido y seguirá siendo fundamento riguroso e ineludible en una gran cantidad de disciplinas y especialidades, lo que muestra la necesidad de su inclusión y su potencial en muchas propuestas curriculares dentro de la formación académica a todos los niveles. Sin embargo, diversos autores la refieren como la menos popular de las disciplinas en casi cualquier plan de estudios. Esta falta de popularidad y, en consecuencia, las dificultades que se presentan tanto en su aprendizaje como en su enseñanza, tienen múltiples razones. Lo cierto es que está presente una significativa

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aversión de los estudiantes hacia las matemáticas, generando así los bajos niveles de rendimiento en esta área del conocimiento. En el presente artículo, se expondrán dos elementos considerados determinantes para el aprendizaje de las matemáticas. En primer lugar, la comunicación efectiva y el lenguaje como foco para que profesores y estudiantes lleguen a acuerdos sobre los significados de los objetos matemáticos compartidos y, en segundo lugar, el dominio afectivo, relacionado con las creencias, las actitudes y las emociones, que determinan la valoración subjetiva que cada individuo demuestra cuando se enfrenta a procesos de aprendizaje relacionados con las matemáticas y que son, precisamente, los responsables de las reacciones, más negativas que positivas, que expresan.


COMUNICAÇÃO EFICAZ E DOMINIO AFETIVO NO APRENDIZADO DE MATEMÁTICAS

RESUMO

A matemática foi e seguirá sendo base rigorosa e inevitável em uma grande quantidade de disciplinas e especialidades, o que mostra a necessidade da sua inclusão e seu potencial em muitas propostas curriculares dentro da formação acadêmica em todos os níveis. Porém, diversos autores se referem a ela como a menos popular das disciplinas em quase qualquer plano de estudos. Esta falta de popularidade e, em consequência, as dificuldades que se apresentam tanto no aprendizado quanto no ensino, tem múltiplas razões. O certo é que está presente uma significativa aversão dos alunos para as matemáticas, gerando assim os baixos níveis de rendimento nesta área do conhecimento. No presente artigo, se irá expor dois elementos considerados determinantes para o aprendizado da matemática. No primeiro lugar, a comunicação efetiva e a linguagem como foco para que professores e alunos cheguem a acordos sobre os significados dos objetos matemáticos compartilhados e, em segundo lugar, o domínio afetivo, relacionado com as crenças, as atitudes e as emoções, que determinam a valorização subjetiva que cada indivíduo demonstra quando enfrenta processos de aprendizado relacionados com a matemática e que são, precisamente, os responsáveis das reações mais negativas que positivas, que expressam.


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1. INTRODUCTION

Initiatives that focus on improving the learning process of mathematics will always be timely, fundamentally due to the conception of this disciplinary area as a complex science and because of the additional ingredient of the nature of the learner, with the consequent subjective assessment that that apprentice shows.

In this context, the understanding of mathematical thinking and the didactic considerations that derive from it is considered relevant, understanding that the construction of meanings must occur in learning environments in which complex and contextualized elements are intentionally incorporated; where there is observation, manipulation, and collaboration; in which the apprentice is allowed to articulate the new with the already known and where reflection is possible. However, this described productive exchange of learning will have, as a transversal axis, effective communication as a powerful mechanism for negotiating mathematical meanings and developing thinking strategies, fundamental competencies to continue learning. Consequently, starting from the fact that the mechanism through which we share ideas, problems, and opinions is language, it should be obvious that ideas, concepts, and mathematical problems are also transmitted through a language that we call technical and whose ignorance is usually the cause of the greatest difficulties in learning mathematics.

Besides the intrinsic complexity of mathematics, attention must be focused on the nature of the individual who learns. For this, the factors that determine the subjective assessment that each apprentice demonstrates when facing mathematics or processes linked to it could be delimited. The truth is that these factors are deeply rooted in the subjects and are precisely those responsible for the reactions they have to objects of mathematical knowledge (Martínez, 2005). We then enter the sphere of the affective domain, which includes aspects of a different nature that can be feelings, emotions, beliefs, attitudes, values, and appreciations (Gómez-Chacón, 2000).

2. CONSTRUCTIVIST CONCEPTION OF MATHEMATICAL THINKING

One of the central challenges of education is the construction, by the individual, of contextualized, culturally accepted, or socially valid meanings, so that a global constructivist theory becomes relevant.

Constructivism, conceived as an epistemological position, is based on the active role of the learner and the conception of knowledge as a permanent process of adaptation of the subject to his environment. This has generated at least three basic constructivist positions. The first, called simple or naive constructivism, for those who only accept the first assumption; the second, radical constructivism, for those who accept both assumptions, and, a third position, known as social constructivism, for those who emphasize the importance of the central role that cognitive conflict should have in the construction of knowledge (D’Amore, 2004).
In a very general conception of learning within the framework of constructivist theory, faced with a new object of knowledge, each individual makes use of their cognitive schemes and structures, their perceptions, and their previous knowledge, to build new learning. In this complex process, their schemes or structures are consolidated, their perceptions are reformulated, and their knowledge acquires new meanings (Boggino, 1998). However, this is clearly a simple or naive position, since the exchange between the apprentice and whoever mediates that learning is not present.

Now, by conceiving knowledge as a permanent process of adaptation of the subject to its environment, the vision of radical constructivism is assumed, which, besides, by incorporating that conflict that arises from the confrontation between our previous conceptions about a certain idea, object, or concept and, a new stimulus that does not fit into the scheme that we have built or that differs from another with which we have the opportunity to share learning spaces, we are facing the position of social constructivism. In short, it is considered global constructivism that incorporates the learner in his leading role, his interaction with the environment, and the cognitive conflicts derived from this interaction.

Up to this point, the conception expressed in the preceding paragraph allows us to describe how learning occurs, but it does not state how the accompaniment or mediation process should be on the part of whoever appears as a teacher. One of the elements to consider in this enriching relationship that must be established between student and teacher is that each individual has a very particular style of learning and that these styles or modes condition the way of perceiving the environment, processing the received information, and determine how they communicate with the environment that surrounds them.

Once the educational phenomenon is fully understood, it is appropriate to focus attention on the constructivist conception of mathematical thinking. For this, it is important to return from genetic psychology (Piaget in Boggino, 1998), the distinction of three types of knowledge: physical, social, and logical-mathematical, and to highlight the difference between them. In particular, logical-mathematical knowledge can be conceived as a collection of mental relationships built by the subject in his interaction with the objects of knowledge, so it does not exist by itself in reality. From this perspective, it is up to each individual to build relationships of seriation, classification, order, inclusion, comparison, analysis, synthesis, reciprocity, and transitivity, among others, for which knowledge is not the product of the direct action of the learner on the objects, but the construction that he reflexively makes (Boggino, 1998). Physical knowledge, for its part, is that which is acquired through the manipulation of objects that are part of the environment, of physical reality, and the subject is empirically incorporated. Finally, social knowledge is arbitrary and is directly related to social conventions and consensuses and the source of this type of knowledge is others, with whom we share learning spaces.

In light of the foregoing, it would be convenient for the didactics of mathematics to be determined by the type of knowledge that corresponds to it, that is, the logical-
mathematical. In this sense, due to the complexity of any mathematical notion, where we can identify elements such as language (notations), situations (problems), actions (how to solve problems), concepts (definitions), propositions (properties), and arguments (demonstrations), the need for a didactic model that incorporates this constructivist conception of mathematical thinking becomes evident.

3. EFFECTIVE COMMUNICATION AND LANGUAGE IN THE LEARNING OF MATHEMATICS

Language, conceived as the human faculty that allows the individual to communicate with himself and with the outside world, implies the handling of a series of codes, symbols, and reference systems organized according to certain laws, to express what the individual experiences in each of the areas of his life.

Through language, human beings build their perception of the world, interact with others, interpret their social environment, develop intellectual potential, and, ultimately, join the time in which they live.

It could be affirmed that man is the product of language if one considers the conception presented by Cadenas (2002), who affirms that language is not only the main means of communication of the human being with the world but that it is also the mechanism through which he thinks and expresses his feelings and ideas.

Now, if language represents for man the only and universal means for his growth and development, it seems reasonable to suppose that it should be considered as the fundamental instrument for the development of any process of learning and knowledge of reality. But, besides, in all activities of an academic nature, both oral and written communication processes are essential, thus establishing a clear relationship between mastery of the language and the success of the student's performance in their learning process. This close relationship between language and other areas of knowledge must be reinforced because, on the one hand, language facilitates the acquisition and transmission of knowledge and, on the other, the other areas of knowledge provide the content without which the development of communication becomes irrelevant and without context (Cárdenas et al., 2001).

Throughout the evolution of knowledge and, especially, in that of specific areas of knowledge, new knowledge, concepts, definitions, and, consequently, new words, symbols, and forms of expression have been generated. Thus, for example, mathematics can be considered a universal language, which implies that everyone should efficiently handle the bases of this language. As such, mathematics offers rules of general use that are essential for the achievement of meaningful learning in this area of knowledge. As has been commented in the preceding paragraphs, its learning causes, in general, significant difficulties for many due, fundamentally, to the fact of having to face a different language than the everyday one. For this reason, attention should be paid to the cause of these difficulties, also considering the enormous power of mathematics in problem solving and decision-making.
For Godino (et al, 2008), the development of a language to express mathematical ideas is essential to communicate, since it is the means of communication with the environment. In particular, within the context of the classroom, teachers and students use language as a tool that allows them to reach agreements on the meanings of shared mathematical objects.

When reflecting on the need to communicate what has been discovered, make it known, and transferable, it becomes clear that it is essential to concur, agree, and share an easy way of communication. This mode of communication includes what we call conventions, which are nothing more than decisions agreed between groups of the same language to understand each other in the use, interpretation, and meaning of each symbol, word, or language elements. In other words, it is implicit that we refer to social knowledge, which, as we mentioned above, is arbitrary and is related to social consensus, but in this case, between people in the specific area. In this sense, in the area of mathematics, its own language has developed and consolidated over the years: the language of mathematics.

Maier (1999), points out some characteristics of mathematical language, which differentiates it from everyday language:

- In mathematical language, directional elements are not accepted unless they are inserted in a reference frame, previously defined. This means, for example, that descriptions of geometric representations in space do not accept terms like "up", "down", "to the left", and so on. A coordinate system is required in which there are no ambiguities.
- All math concepts are ideal. This means that, in a large number of these concepts, there is no relation to real objects, that is, it is not about physical knowledge. They can also define ideal relationships between objects or sets of objects (even if they are real). This ideal character of mathematics means that concepts must be constructed through discourse, the most complete form of which is a definition.
- This idealization that is done in mathematics does not allow ambiguities, therefore, each object, term, or symbol is perfectly defined, is unique, and each particular meaning corresponds to a single object, term, or symbol.
- Mathematical definitions are strict, which means that they must be clearly differentiated from similar terms in everyday language.
- Mathematical language and, consequently, its interpretation is limited to a specific field of application and must always be clearly defined.

Besides the mentioned conceptual characteristics, other elements that can be organized into two large groups are also found within the language of mathematics: the elements of organization and representation of information, on the one hand, and, on the other, the symbolic elements. Regarding the first, you can find a wide range of tools to organize and represent the information that is used in the mathematics. Then there is talk of the use of tables, Euler-Venn diagrams, circular diagrams, bar diagrams, coordinate systems, among others. As for symbolic elements, we refer to the frequent use in mathematics of elements more linked to everyday language. For example, we
must remember that, in mathematics, the use of letters is a very powerful tool for representing variable quantities and making important generalizations.

Obviously, this article is not intended to be exhaustive with all the specific elements, symbols, and conventions of mathematical language, but rather to highlight the relevance of knowing it and thus enable reflection on the importance it has for the achievement of effective communication in this area of knowledge, and, thus, initiate more fluid and meaningful learning processes. So the purpose of considering everyday and mathematical language, deliberately, is to train individuals with management of communicative exchange that allows them to respect the ideas of others, clarity and coherence of the message, affectivity, adapt the message to the context, and learn new meanings.

Traditionally, writing and reading comprehension is considered a skill that is "taught" almost exclusively in the area of language arts, however, teachers in all areas hope that the student will be able to transfer this knowledge to new contexts, with other characteristics and this, from the outset, is not questionable, what should not happen is that students are abandoned in the development of the necessary skills, going beyond transmitting an accumulation of concepts without ensuring their adequate understanding.

4. THE AFFECTIVE DOMAIN IN THE LEARNING OF MATHEMATICS

An element of great importance in the learning of mathematics, beyond what has been exposed so far, are the affects (affects, attitudes, and beliefs) on individuals and their relationship with the successful desired performance.

The affects are determinants of the individual context in which the resources, strategies, and self-control of the learner are linked (Gómez-Chacón, 2000). They also influence the vision that each learner has about themself, altering the relationships that occur with their own cognitive system. Likewise, the affects intervene in the mode of social organization within any learning environment and, without a doubt, can constitute an obstacle to meaningful learning. It is clear that the rigidity and negativity of the beliefs that a student may have about mathematics and its learning, determine the strategies that he uses, which, in many cases, are limited to memorization instead of trying to understand.

Mathematics has been present in almost all the activities of man, without a doubt, so it is almost impossible to find any phenomenon in which its magnificent explanatory power can be evaded. In fact, mathematics has been the rigorous foundation of the vast majority of disciplines, showing its potential in almost all curricular proposals within the academic training of all levels. However, as already mentioned, various authors refer to it as the most unpopular of the disciplines (Martínez, 2005).

The poor popularity that mathematics has traditionally had has greatly disadvantaged both its learning and its teaching. In this sense, some authors (Godino, 2008; Martínez, 2005; Gil et al, 2006) argue that students' aversion to mathematics
supports negative attitudes towards their learning. This is the reason why the
generalized low performance in this area of knowledge forces us to make a deep
reflection on the affective and emotional factors in the mathematical learning process.

If we analyze mathematics as an intellectual activity, we find that its fundamental
characteristic is mathematization, which definition points to the process of building
mathematical models. This results in the sequencing of organization and structuring
processes of the information that appears in a problem, identification of the relevant
mathematical aspects, and the discovery of regularities, relationships, and structures
(García, 2000). Within this process, two stages or moments can be distinguished. A
first moment in which one passes from the real world to the world of symbols
(horizontal mathematization), with the consequently associated processes of
identification, discovery, and transfer, and a second moment, in which situations are
dealt with mathematically and we identify as processes the representation, the use of
models, the formulation, the tests, and the generalizations (vertical mathematization)
(García, 2000).

The mathematization processes have defined, depending on the leverage of each
of the moments (horizontal or vertical), different teaching methodologies. But this
simplification of the teaching of mathematics, that is, associating it exclusively with its
cognitive aspects, would be avoiding the complexity of mathematics as a discipline
and, in part, limiting the complex nature of the individual who, ultimately, is the
learner.

Regarding the complexity of mathematics as a discipline, Andonegui (2004) affirms
that it is possible to visualize it from different perspectives. He highlights that it goes
through epistemological aspects associated with how each mathematical object is
constructed, how it is represented, how it is related to others, and how mathematical
knowledge is validated. For this author, another element that contributes to the
complexity of mathematics is linked to the notion of contents that are not real, such
as quantity, shape, symbol, representation, determination and uncertainty, change,
among others. On the other hand, the historical constructivist aspect allows the
exercise of imagination, intuition, analogy, metaphor, analysis, and synthesis.
Additionally, two aspects that make mathematics a complex science stand out: the
possibility of modeling and application to real situations, and the aesthetic aspect of
symmetries, regularities, generalizations, and singularities.

In addition to the intrinsic difficulty of mathematics, special attention must be paid
to the complexity of the nature of the learner. To begin with, we could try to identify
the factors that determine the subjective assessment that each individual shows when
faced with mathematics or processes related to it. What we know about this is that,
without a doubt, these factors are deeply rooted in the subjects, which makes it even
more difficult to address them from mathematics didactics (Martínez, 2005).

The truth is that this extensive range of feelings and states of mind, which can also
be separated or considered apart from knowledge, is what authors such as Mcleod
(1989, in Gil et al., 2006) define as affective domain, whose basic descriptors can focus on beliefs, attitudes, and emotions (Gil et al., 2006).

Mathematical beliefs are a component of the individual's knowledge in the experience and are defined as subjective knowledge and experiences of the subject, and are referred to both teachers and students. In them, it is possible to distinguish two differentiated elements, on the one hand, the belief of students associated with mathematics itself, in terms of its level of difficulty, its methodologies, or its rules, which is obviously more associated with the educational system, the school context, and classroom activity. But, in addition, another element can be identified as the belief of the subject towards their own ability towards mathematics (Gil et al., 2006).

The attitude towards mathematics has been defined by Bazán (1997) as the phenomenon that involves feelings (affective component), beliefs (the cognitive component), and the tendencies of the subjects to act in a particular way approaching or moving away from the "mathematical" object. For Bazán (1997) the attitude towards mathematics can be seen from the four dimensions that make it up: the skill dimension (confidence of the subject in their mathematical ability), the applicability dimension (assessment made by the subject of the application that mathematics may have), affectivity (associated with liking or disliking mathematics), and anxiety (anxious reactions towards mathematics).

From all the revised definitions of attitudes, a series of important aspects can be inferred that from the perspective of Castro (2002) go through accepting that attitudes are acquired; they suppose a high affective and emotional load; they imply acceptance or rejection of the stimulus and are considered valuations that go beyond the descriptions of those stimuli; they represent chosen responses to certain values; they are understood as multi-dimensional structures; they are subjective experiences and constitute stable learning, so they can be encouraged, modified, and learned. This last characteristic means that it is possible to achieve changes in the attitude towards mathematics through motivating elements that allow students to carry out a possible reevaluation of their attitudes towards mathematics. This is where knowing the power of mathematics as a modeling mechanism of real situations, its scope as a developer of generic thinking strategies, as well as its connection or link with the lives of human beings, their problems, and their contexts play such an important role.

As a third descriptor of the affective domain are emotions, considered responses organized above psychological systems, which have a positive or negative load of meaning for the individual. Therefore, emotions are the complex result of learning, social influence, and interpretation (Gómez-Chacón, 2000).

Gómez-Chacón (2003) points out other relevant aspects of the affective dimension of learning mathematics. The first of them is linked to the understanding of affect as a system of representation in individuals, that is, it cannot be considered isolated or separated from cognition, which means that it conditions the cognitive appreciation of learning objects. The second aspect is associated with the fact that affects have both a biological and social basis because the social context has a strong influence on beliefs.
since they are acquired through processes of cultural transmission. The last aspect that this expert on the subject of affects in mathematics learning highlights is the distinction between local and global affect, indicating the scope that this affect can give to an individual as part of a certain social group.

But there is even more, understanding the teaching-learning process as a complex whole in which several actors participate, it is worth mentioning the affective domain but from the teacher's point of view. In this sense, Gómez (1995) affirms that the quality of the teacher is seriously influenced by his epistemological vision regarding the mathematical objects of study. The author suggests that this vision may be located on a continuum whose extremes are perfectly differentiated: the closed vision that is the paradigm to be transmitted and the open vision of an activity for solving problems.

The closed epistemological vision is strengthened in the position of seeing the subject of mathematical study as a set of truths and relegates the student to a totally passive position as a receiver of those truths (Gómez, 1995). This implies, for the students of teachers with this epistemological vision, the consolidation of negative beliefs towards mathematics. This means that the student cannot create spaces to distinguish the learning elements necessary for the construction of permanent structures that allow him to approach new mathematical topics and objects. This vision is totally focused on content and not on learning processes.

For its part, the open epistemological vision in which the objects of mathematical study are conceived as activities in the search for solutions to problems, allows students to develop the necessary capacities to creatively use knowledge, facilitating a more concrete and applied vision of mathematical objects. In this way, the formation of more constructive beliefs about mathematics is encouraged (Gómez, 1995).

It is clear that affects and, with them, beliefs, attitudes, and emotions, are fundamental factors in the construction of mathematical meanings and, therefore, in the performance of learners, a didactic proposal in this area cannot forget the consideration of the affective aspect in it. For this, elements ranging from understanding and mastery of mathematical content by the teacher, as a mechanism to ensure an undistorted vision, in the students, to the incorporation of elements, in the curricula, related to cognition and affect, mathematics as cultural knowledge, as well as the management of the student's self-concept, must be taken into account.

5. CONCLUSIONS

Learning mathematics faces important challenges since in any mathematical notion we can identify elements such as language (notations), situations (problems), actions (how to solve problems), concepts (definitions), propositions (properties), and arguments (demonstrations), so the need for a didactic model that incorporates this constructivist conception of mathematical thinking is evident.

For its part, the development and understanding of a language to express mathematical ideas are essential to communicate, since it is the means of exchange
with the environment. In particular, within the context of the classroom, teachers and students use language as a tool that allows them to reach agreements on the meanings of shared mathematical objects. For this reason, when reflecting on the need to communicate what has been discovered, make it known, and transferable, it is evident that it is essential to concur, agree, and share an easy way of effective communication.

The beliefs, attitudes, and emotions of both the teacher and the student condition how both, in their own way and with their particular expectations, approach mathematical objects in a learning space, which is why awareness of this challenge is essential to investigate, in students, their particular beliefs and attitudes towards mathematical objects of study, with the certainty that it is possible that they may be modified to overcome the barriers that the learning of mathematics has traditionally had.

6. REFERENCES


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